

$\rho ::=$	エスケープの有無 (束を構成)
$true$	エスケープするかもしれない ( $\top$ )
$false$	決してエスケープしない ( $\perp$ )
$G ::=$	現在のフレームよりもグローバルな array か否か (束を構成)
$true$	グローバルな array かもしれない ( $\top$ )
$false$	決してグローバルな array ではない ( $\perp$ )
$\tau ::=$	型
$\pi$	プリミティブ型
$(\bar{\tau} \rightarrow \tau)_\rho$	関数型
$(\tau_1 \times \dots \times \tau_n)_\rho$	組型
$(\tau \text{ array})_{\rho, G}$	配列型
$e ::=$	式
$c$	定数
$op(e_1, \dots, e_n)$	プリミティブ演算
$\text{if } e_1 \text{ then } e_2 \text{ else } e_3$	条件分岐
$\text{let } x = e_1 \text{ in } e_2$	変数定義
$x$	変数の読み出し
$\text{let rec } \rho \ x \ \bar{y} = e_1 \text{ in } e_2$	再帰関数定義
$e_1 \ \bar{e}_2$	関数呼び出し
$(e_1, \dots, e_n)_\rho$	組の作成
$\text{let } (x_1, \dots, x_n) = e_1 \text{ in } e_2$	組の読み出し
$\text{Array.create}_\rho \ e_1 \ e_2$	配列の作成
$e_1.(e_2)$	配列の読み出し
$e_1.(e_2) \leftarrow e_3$	配列への書き込み

図 1: エスケープ解析用に拡張された Min-Caml の構文。 $\bar{x}$  等は  $x_1, \dots, x_n$  等の略。 $\rho$  はコンパイラがエスケープ解析により補う

$$Globalize(x_1 : \tau_1, \dots, x_n : \tau_n) = x_1 : Globalize(\tau_1), \dots, x_n : Globalize(\tau_n)$$

$$Globalize((\bar{\tau} \rightarrow \tau)_\rho) = (Globalize(\bar{\tau}) \rightarrow Globalize(\tau))_\rho$$

$$Globalize((\tau_1 \times \dots \times \tau_n)_\rho) = (Globalize(\tau_1) \times \dots \times Globalize(\tau_n))_\rho$$

$$Globalize((\tau \text{ array})_{\rho,G}) = (Globalize(\tau) \text{ array})_{\rho,true}$$

$$Globalize(\tau) = \tau \quad (\text{上記以外})$$

$$Escapes((\bar{\tau} \rightarrow \tau)_\rho) = \rho$$

$$Escapes((\tau_1 \times \dots \times \tau_n)_\rho) = \rho$$

$$Escapes((\tau \text{ array})_{\rho,G}) = \rho$$

$$Escapes(\tau) = true \quad (\text{上記以外})$$

図 2: エスケープ解析用の補助関数

$$\frac{c \text{ は } \pi \text{ 型の定数}}{\Gamma \vdash c : \pi} \quad \frac{\Gamma \vdash e_1 : \pi_1 \quad \dots \quad \Gamma \vdash e_n : \pi_n}{op \text{ は } \pi_1, \dots, \pi_n \text{ 型の値を受け取って } \pi \text{ 型の値を返すプリミティブ演算}}{\Gamma \vdash op(e_1, \dots, e_n) : \pi}$$

$$\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \tau} \quad \frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_2}$$

$$\frac{\Gamma(x) = \tau \quad \Gamma \vdash x : \tau}{\Gamma \vdash x : \tau} \quad \frac{Globalize(\Gamma, x : (\bar{\tau} \rightarrow \tau)_\rho, \bar{y} : \bar{\tau}) \vdash e_1 : \tau \quad \Gamma, x : (\bar{\tau} \rightarrow \tau)_\rho \vdash e_2 : \tau' \quad true \sqsubseteq Escapes(\tau)}{\Gamma \vdash \text{let rec } \rho \ x \ \bar{y} = e_1 \text{ in } e_2 : \tau'} \quad \frac{\forall z \in \mathcal{FV}(e_1) \setminus \{x, \bar{y}\}. \rho \sqsubseteq Escapes(\Gamma(z)) \quad \Gamma \vdash e_1 : (\bar{\tau} \rightarrow \tau)_\rho \quad \Gamma \vdash \bar{e}_2 : \bar{\tau}}{\Gamma \vdash e_1 \ \bar{e}_2 : \tau}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \dots \quad \Gamma \vdash e_n : \tau_n \quad \rho \sqsubseteq Escapes(\tau_1) \quad \dots \quad \rho \sqsubseteq Escapes(\tau_n)}{\Gamma \vdash (e_1, \dots, e_n)_\rho : (\tau_1 \times \dots \times \tau_n)_\rho} \quad \frac{\Gamma \vdash e_1 : (\tau_1 \times \dots \times \tau_n)_\rho \quad \Gamma, x_1 : \tau_1, \dots, x_n : \tau_n \vdash e_2 : \tau}{\Gamma \vdash \text{let } (x_1, \dots, x_n) = e_1 \text{ in } e_2 : \tau}$$

$$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \tau \quad \rho \sqsubseteq Escapes(\tau)}{\Gamma \vdash \text{Array.create}_\rho \ e_1 \ e_2 : (\tau \text{ array})_{\rho,G}} \quad \frac{\Gamma \vdash e_1 : (\tau \text{ array})_{\rho,G} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1.(e_2) : \tau}$$

$$\frac{\Gamma \vdash e_1 : (\tau \text{ array})_{\rho,G} \quad \Gamma \vdash e_2 : \text{int} \quad \Gamma \vdash e_3 : \tau \quad G \sqsubseteq Escapes(\tau)}{\Gamma \vdash e_1.(e_2) \leftarrow e_3 : \text{unit}}$$

図 3: エスケープ解析用の型付け規則

プログラム:

$$\text{let rec } f \ x = (\text{let } p = (x, x) \text{ in let } q = (p, x) \text{ in } q) \text{ in } f \ 3$$

型付け:

$$\frac{\frac{\frac{f : \tau_f, x : \mathbf{int} \vdash x : \mathbf{int}}{f : \tau_f, x : \mathbf{int} \vdash (x, x)_{\rho_p} : \tau_p} \quad \frac{f : \tau_f, x : \mathbf{int} \vdash x : \mathbf{int}}{f : \tau_f, x : \mathbf{int}, p : \tau_p \vdash p : \tau_p} \quad \frac{f : \tau_f, x : \mathbf{int}, p : \tau_p \vdash x : \mathbf{int}}{f : \tau_f, x : \mathbf{int}, p : \tau_p \vdash (p, x)_{\rho_q} : \tau_q} \quad \dots}{f : \tau_f, x : \mathbf{int}, p : \tau_p \vdash \text{let } q = (p, x)_{\rho_q} \text{ in } q : \tau_q} \quad \dots}{f : \tau_f, x : \mathbf{int} \vdash \text{let } p = (x, x)_{\rho_p} \text{ in let } q = (p, x)_{\rho_q} \text{ in } q : \tau_q} \quad \dots}{\emptyset \vdash \text{let rec }_{\rho_f} f \ x = (\text{let } p = (x, x)_{\rho_p} \text{ in let } q = (p, x)_{\rho_q} \text{ in } q) \text{ in } f \ 3 : \tau_q}$$

ただし  $\tau_p$  は  $(\mathbf{int} \times \mathbf{int})_{\rho_p}$ 、 $\tau_q$  は  $(\tau_p \times \mathbf{int})_{\rho_q}$ 、 $\tau_f$  は  $(\mathbf{int} \rightarrow \tau_q)_{\rho_f}$ 、の略記

制約:

$$\begin{aligned} \rho_p &\sqsubseteq \text{Escapes}(\pi) & \rho_p &\sqsubseteq \text{Escapes}(\pi) & (\text{組 } p) \\ \rho_q &\sqsubseteq \text{Escapes}(\tau_p) & \rho_q &\sqsubseteq \text{Escapes}(\pi) & (\text{組 } q) \\ \text{true} &\sqsubseteq \text{Escapes}(\tau_q) & & & (\text{返り値}) \end{aligned}$$

Simplify された制約:

$$\begin{aligned} \rho_q &\sqsubseteq \rho_p \\ \text{true} &\sqsubseteq \rho_q \end{aligned}$$

反復アルゴリズムによって求まる解:

$$\begin{aligned} \rho_p &= \text{true} \\ \rho_q &= \text{true} \\ \rho_f &= \text{false} \end{aligned}$$

図 4: エスケープ解析の例 1

プログラム:

let  $a = \text{ref } (1, 2)$  in let rec  $f () = \text{let } b = \text{ref } (3, 4)$  in  $a := (5, 6); b := (7, 8)$  in  $f ()$

ただし  $\text{ref } e$  は  $\text{Array.create } 1 \ e$ 、 $e_1 := e_2$  は  $e_1.(0) \leftarrow e_2$ 、 $e_1; e_2$  は  $\text{let } \_ = e_1$  in  $e_2$ 、の略記

型付け:

$$\frac{\frac{\frac{\vdots}{\emptyset \vdash (1, 2)_{\rho_{12}} : \tau_{12}}}{}{\emptyset \vdash \text{ref}_{\rho_a}(1, 2)_{\rho_{12}} : \tau_{a-f}} \quad \frac{\frac{\frac{\vdots}{AT \vdash (3, 4)_{\rho_{34}} : \tau_{34}}}{AT \vdash \text{ref}_{\rho_b}(3, 4)_{\rho_{34}} : \tau_{b-f}} \quad \frac{\frac{\vdots}{AT, B \vdash (5, 6)_{\rho_{12}} : \tau_{12}}{AT, B \vdash a := (5, 6)_{\rho_{12}} : \text{unit}} \quad \frac{\frac{\vdots}{AT, B \vdash (7, 8)_{\rho_{34}} : \tau_{34}}{AT, B \vdash b := (7, 8)_{\rho_{34}} : \text{unit}}}{AT, B \vdash a := (5, 6)_{\rho_{12}}; b := (7, 8)_{\rho_{34}} : \text{unit}}}{AT \vdash \text{let } b = \text{ref}_{\rho_b}(3, 4)_{\rho_{34}} \text{ in } a := (5, 6)_{\rho_{12}}; b := (7, 8)_{\rho_{34}} : \text{unit}} \quad \dots}{AT \vdash \text{let rec } \rho_f \ f () = \text{let } b = \text{ref}_{\rho_b}(3, 4)_{\rho_{34}} \text{ in } a := (5, 6)_{\rho_{12}}; b := (7, 8)_{\rho_{34}} \text{ in } f () : \text{unit}}}{\emptyset \vdash \text{let } a = \text{ref}_{\rho_a}(1, 2)_{\rho_{12}} \text{ in let rec } \rho_f \ f () = \text{let } b = \text{ref}_{\rho_b}(3, 4)_{\rho_{34}} \text{ in } a := (5, 6)_{\rho_{12}}; b := (7, 8)_{\rho_{34}} \text{ in } f () : \text{unit}}$$

ただし  $\tau_{12}$  は  $(\text{int} \times \text{int})_{\rho_{12}}$ 、 $\tau_{34}$  は  $(\text{int} \times \text{int})_{\rho_{34}}$ 、

$\tau_a$  は  $(\tau_{12} \text{ ref})_{\rho_a, G_a}$ 、 $\tau_{a.t}$  は  $(\tau_{12} \text{ ref})_{\rho_a, \text{true}}$ 、 $\tau_b$  は  $(\tau_{34} \text{ ref})_{\rho_b, G_b}$ 、 $\tau_{b.t}$  は  $(\tau_{34} \text{ ref})_{\rho_b, \text{true}}$ 、

$A$  は  $a : \tau_a$ 、 $AT$  は  $a : \tau_{a.t}$ 、 $B$  は  $b : \tau_b$ 、 $BT$  は  $b : \tau_{b.t}$ 、の略記

制約:

$$\begin{aligned} \rho_{12} &\sqsubseteq \text{Escapes}(\pi) & \rho_{12} &\sqsubseteq \text{Escapes}(\pi) & (\text{組 } (1, 2)) \\ \rho_{34} &\sqsubseteq \text{Escapes}(\pi) & \rho_{34} &\sqsubseteq \text{Escapes}(\pi) & (\text{組 } (3, 4)) \\ \rho_{12} &\sqsubseteq \text{Escapes}(\pi) & \rho_{12} &\sqsubseteq \text{Escapes}(\pi) & (\text{組 } (5, 6)) \\ \rho_{34} &\sqsubseteq \text{Escapes}(\pi) & \rho_{34} &\sqsubseteq \text{Escapes}(\pi) & (\text{組 } (7, 8)) \\ \rho_a &\sqsubseteq \text{Escapes}(\tau_{12}) & & & (\text{参照 a}) \\ \rho_b &\sqsubseteq \text{Escapes}(\tau_{34}) & & & (\text{参照 b}) \\ \text{true} &\sqsubseteq \text{Escapes}(\tau_{12}) & & & (\text{参照 a への代入}) \\ G_b &\sqsubseteq \text{Escapes}(\tau_{34}) & & & (\text{参照 b への代入}) \\ \rho_f &\sqsubseteq \text{Escapes}(\tau_{a-f}) & & & (\text{関数 f の自由変数}) \\ \text{true} &\sqsubseteq \text{Escapes}(\pi) & & & (\text{関数 f の戻り値}) \end{aligned}$$

Simplify された制約:

$$\begin{aligned} \rho_a &\sqsubseteq \rho_{12} \\ \rho_b &\sqsubseteq \rho_{34} \\ \text{true} &\sqsubseteq \rho_{12} \\ G_b &\sqsubseteq \rho_{34} \\ \rho_f &\sqsubseteq \rho_a \end{aligned}$$

反復アルゴリズムによって求まる解:

$$\begin{aligned} \rho_{12} &= \text{true} \\ \rho_{34} &= \text{false} \\ \rho_a &= \text{false} \\ \rho_b &= \text{false} \\ \rho_f &= \text{false} \end{aligned}$$

図 5: エスケープ解析の例 2